STAT 165/265 HW 10

March 21, 2024

Due Friday, April 5th at 11:59pm

Martingales

Graded on accuracy
Expected completion time: 60 minutes

Recommended reading: Chapter 16 of Prof. Steinhardt's *Forecasting*

Practice with Martingales

Consider the setup from lecture where we have forecasts from Monday and Tuesday for whether it will rain on Friday. Borrowing notation from the textbook (linked above), let π_1 and π_2 denote the proportion of the time it rains on Friday in p_1 and p_2 proportion of the weeks, respectively.

For each scenario below, explain whether it possibly represents well-calibrated beliefs. In other words, explain whether it satisfies the Martingale property.

1. Suppose we always believe on Monday that the probability of rain on Friday is 0.5. But 3/4 of the time we increase this probability to 0.75 on Tuesday, and 1/4 of the time we decrease it to 0.25.
2. Suppose we always believe on Monday that the probability of rain on Friday is 0.45. But 1/8 of the time we increase this probability to 0.8 on Tuesday, and 7/8 of the time we decrease it to 0.4.

Properties of Martingales

Review this textbook section on the properties of martingales. Consider the following ordered sequence of probabilities representing a forecaster's belief at different points in time:

$$0.96, 0.4, 0.02, 0.03, 0.76, 0.97, 0.96, 0.04$$

1. Using notation from the textbook, let $a = 0.05, b = 0.95$. How many "crossings" does the sequence have?
2. Use Doob's upcrossing lemma to find a bound on how likely this sequence would be to have this many (or more) crossings, provided that it is a martingale.

Kelly Betting

Expected completion time: 20 minutes

1. **Optimal Betting Fraction for a Single Bet**:
   Given a single betting opportunity where the probability of winning is 0.6 (60%), and if you win, you receive twice the amount you bet (net gain of 1x the bet amount), calculate the optimal fraction of your current wealth that you should bet according to the Kelly Criterion.
2. **More Kelly Criterion Betting Practice**:
   Given a single betting opportunity where the probability of winning is $\frac{2}{3}$ (66.%), and if you win, you gain 80% more than the amount you bet. According to the Kelly Criterion, what fraction of your wealth should you bet on each event to maximize the expected logarithm of your wealth over time?
3. **To Bet or Not to Bet, the Story of Zero Edge**:
   Given a single betting opportunity where the probability of winning is 0.5 (50%), and if you win, you receive twice the amount you bet (net gain of 1x the bet amount), calculate the optimal fraction of your current wealth that you should bet according to the Kelly Criterion.

Predictions

Expected completion time: 90 minutes
Graded on accuracy as part of the class forecasting competition

Make and submit predictions to the questions on this Google Form:
https://forms.gle/5Qzdn87RwVhoKgmJ7.

Be sure to follow the format described at the top of the form. For each question, you will submit a mean and inclusive 80% confidence interval or a probability (whichever the question asks for). We provide cells on the Google form for you to type out your reasoning (1-2 paragraphs), which you should submit to Gradescope with the rest of this assignment. For questions 1-3, your prediction (but not the explanation) will appear on the public leaderboard.

Final Project Progress Check

Expected completion time: 45 minutes
By the deadline for this homework, we expect your group to have made some progress on your final project. In 100 ($\pm$ 20) words, tell us about what your group has accomplished so far and what your contributions are so far.

Post-mortem

Complete this homework's post-mortem, which is a separate assignment on Gradescope.

[STAT 265 only] None this week